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# Statistical Evaluation of Compressive Strength in High Performance Concrete (HPC) with Steel Fiber Addition

**Abdulhameed Umar Abubakar***

Department of Civil Engineering, Modibbo Adama University of Technology, Yola, Nigeria
*Corresponding E-mail: abdulhameed.umar@mautech.edu.ng

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**ABSTRACT.** *In this study, statistical analysis of High performance concrete (HPC) with steel fiber addition at 0.50, 0.75, 1.00, 1.25, & 1.50 % was evaluated each with a sample size (N) of 50 using 100 mm side cubes. Normality test and screening was done on the data set, frequency histogram with superimposed normal distribution curve plotted, as well as P-value compared with the significance level. Other parameters investigated were confidence interval and probability plot. It was seen that the data does not follow the normal distribution because the data float around the ideal normal curve, P-value is less than the significance level and Anderson-darling is high. An attempt was made to transform the data set for Goodness of fit using Johnson Transformation, and it was seen that the P-value significantly improved. Overall, an improvement in the mean compressive strength is observed with increase in fiber addition by utilizing large sample size.*

**Keywords:** High performance concrete, steel fiber-reinforced concrete, statistical analysis, compressive strength, probability distribution, P-value.

## 1. INTRODUCTION

In Neville [1], HPC is defined as a concrete having a compressive strength in excess of 80 MPa with the following ingredients: good quality aggregates; cement content in the range of 450 – 550 kg/m$^3$; silica fume generally 5 – 15 % by mass of the total cementitious materials or sometimes fly ash or ground granulated blast furnace slag; and always a superplasticizer, the dosage which is very high resulting in a decrease in water content. The low water-cement ratio (always below 0.35) is necessitated from the desire to have a mix that is compact and durable. Significant improvement in mechanical properties of concrete with steel fiber addition have been recorded in recent years. The work of Robins *et al.*, [2] reported that mechanical properties of FRC depends on the characteristics of the concrete matrix, geometry and type of the fibers, which govern the bond mechanism that exist between them and the concrete. It is known that the ultimate strength of concrete depends on the behavior of the concrete a point very close to onset of rapid crack propagation (failure load). In concrete with fiber addition, cracks in the direction of the fibers are intercepted resulting in high carrying capacity of the concrete.

In high-strength concretes on the other hand, it is known that the linear portion of the stress-strain curve extends up to 80 % of the ultimate strength, sometimes even more according to Neville [1] cited in [3]. Recently, Abubakar [4], reported that the linear portion extends from 85 – 91 % of the ultimate in concrete with steel fiber addition from 0.5 – 2.0 %. Some researchers [5] attributed this delay in the formation of microcracks to the non-existence of microcracking at initial or ascending portion of the curve resulting in explosive failure, due to very high brittleness. However, with fiber addition, resistance of concrete to crack increases considerably [6]. This is because fibers bridge the gap between adjacent microcracks which serves to delay the initiation of cracks and limit the propagation, producing a small crack opening displacement. They control the opening as well as the propagation of the cracks resulting in very high fracture energy from pull-out mechanism [7]. According to Neves and Fernande de Almeida [8] reinforcement provided by fibers work at both micro and macro level, where in the former arresting the propagation of microcracks leading to higher Compressive strength, $f_c$. On the other hand, in the latter, restricting the crack opening, and increased energy absorption of the concrete is observed.

There is conflicting information in the literature regarding the behavior of $f_c$ in concrete with steel fiber addition from normal strength concrete (NSC) to ultra-high performance concrete (UHPC). In ACI 544.1R [9], it is reported that the $f_c$ of concrete with fiber addition is slightly improved for up to 1.5 % addition by volume of fibers. Other studies that had similar findings include Afroughsabet and Ozbakkaloglu [10], and Afroughsabet *et al.,* [11] working with recycled aggregate concrete. The work of Corinaldesi and Nardinocchi [12] studied the influence of coating and expansive agent on different types of fibers. They reported that the 28-day $f_c$ irrespective of the coating and amount of expansive agent used resulted in a 26 – 40 % increase in strength for steel fibers. In Kazemi *et al.,* [13], an increase in $f_c$ from fiber addition of 0 – 1.6 % of concrete volume is observed. Also, Wu *et al.*, [14] reported that fiber hybridization results in improvement of static $f_c$ by as much as 20 – 46 % in UHPC. A decrease in the value of $f_c$ has also been reported which has been attributed in one study to difficulty in achieving compaction of the concrete [15]. Similarly, Yoo *et al.,* [16], and Le Hoang and Fehling [17] observed and reported slight decrease in $f_c$ of UHPC with steel fiber addition. The work of Aydin [18-19] experimented on staple wire reinforcement in high-volume fly ash cement paste composite reported a decline in compressive strength at an age of 28 days at volume fraction above 3%. Also, no trend at all has been reported by [20-21].

In this regard, $f_c$ determination is very important especially when it comes to the investigation of the failure mechanism in HPC with steel fiber addition. There are a number of factors that affects this property such as type and amount of cement, water content, amount of additives, aggregates (types, properties, sizes etc.), mixing process, compaction, and curing process [22-23]. A study by Abubakar [4] on the influence of specimen dimension on $f_c$ with 100 mm side cubes, 150 mm side cubes, and 150 x 300 mm cylinders, showed a retrogression and stagnation in strength on 100 mm side cubes. The 150 mm sides cubes and the cylinder showed an increase in strength with fiber addition. Results of the smaller sized cubes is similar to that of a pilot study conducted by Abubakar *et al.,* [21] on the effect of curing regimes on optimized high performance steel fiber concrete mixes.

It is on this premise that an attempt is made here to apply statistical methods to evaluate those variabilities if any in specimens of 100 mm cube sides with large sample size ($n > 30$) sample size. Previous works such as Tirkolaei and Bilsel [24] and Mosaberpanah and Eren [25] have utilized response surface methodology (RSM) in design of experiment (DOE) for mixture optimization.

## 2. EXPERIMENTAL PROCEDURE

Blast-furnace Slag Cement CEM II/B-S 42.5 N in conformity with ASTM C 595 [26] was utilized with a specific gravity of 3.15; silica fume was added at 10% of the cement content with 82 % content of $SiO_2$ (Table 1), and tap water utilized conformed with BS EN 1008 [27]. High range water reducer (HRWR) utilized was GLENIUM 27 conforming with ASTM C 494 [28] ether basis brown in color with a density of 1,023 – 1,063 kg/lt color content <0.1 % and alkali content <3%.

**Table-1:** Chemical analysis

| Material | $SiO_2$ | $Al_2O_3$ | $Fe_2O_3$ | CaO | MgO | $SO_3$ | *LOI | **I.R. | #S.G. ($g/cm^3$) |
|---|---|---|---|---|---|---|---|---|---|
| **Cement(%)** | 18.72 | 4.04 | 2.56 | 60.44 | 2.00 | 2.24 | 10.88 | 0.10 | 3.15 |
| **S.F. (%)** | 82.20 | 0.50 | 0.42 | 1.55 | 0.00 | 3.03 | 5.66 | - | 2.2 |

*Loss on ignition; ** Insoluble Residue; #Specific Gravity

Aggregates used were crushed limestone rock conforming to the specification of ASTM C 33 [29] and sieve analysis was conducted in accordance with ASTM C 136 [30] and the result presented.

**Table-2:** Sieve analysis of aggregates

| **Sieve sizes (mm)** | 12.5 | 9.5 | 4.75 | 2.36 | 1.18 | 0.6 | 0.3 | 0.15 | 0.075 |
|---|---|---|---|---|---|---|---|---|---|
| **% passing coarse** | 100 | 100 | 9.4 | 0.2 | 0.2 | - | - | - | - |
| **% passing fine** | - | - | 100 | 82 | 48 | 29 | 17 | 7 | 3 |

Relative density (SSD) performed for both fine and coarse aggregates were based on ASTM C 127 [31] and ASTM C 128 [32] and were 2.68 and 2.65 respectively, also absorption (%) determined based on the same standards were 3.0 % and 0.7 %. Bulk density was based on ASTM C 29 [33] were 2083 $kg/m^3$ and 1203 $kg/m^3$. Voids in aggregates were 25 % and 50 % respectively for fine and coarse aggregates. The percentage of materials finer than 75µm to ASTM C117 [34] was 3%. Hooked end steel fiber were utilized with the following volume fractions 0.5, 0.75, 1.0, 1.25, & 1.50 % by volume of concrete (39.25, 58.88, 78.50, 98.125, & 117.75 $kg/m^3$, respectively). Steel fibers conformed to ASTM A820 [35] with the properties presented in Table 3.

**Table-3:** Dimensions and tensile strength of steel fibers

| Length (mm) | Diameter (mm) | L/d Ratio | Tensile Strength (MPa) |
|---|---|---|---|
| 30 | 0.50 | 60 | 1250 |

l/d: Length-to-diameter ratio

Batching was done based on the proportioning presented in Table 4, and mixing operation was done as prescribed by the manufacturer with the fibers which are stacked in a fibrillated bundle of water soluble glue placed last by distribution in small amount to avoid balling. Immediately upon contact with moisture they were dispersed but the mixing time was relatively longer for volume percentages from 1.25 % and above.

**Table-4:** Mix design utilized for HPC

| Material | Cement 42.5 N (Slag) | Water | Coarse (10mm max) | Fine (5mm max) | Silica Fume | HRWR | $f_c$ (MPa) |
|---|---|---|---|---|---|---|---|
| Quantity ($kg/m^3$) | 470 | 165 | 1050 | 700 | 47 | 14 | 86 |

Concrete compressive strength was tested at age of 28 days for prismatic specimens when tested based on BS EN 12390 – 3 [36] with fifty 100 mm cubic prisms at a loading rate of 0.5 MPa/s with a 3,000 kN capacity machine. The data was collected, arranged, and sorted from the smallest to the largest (as is presented in the Appendix), subsequently, Minitab 18 [37] statistical software which can be obtained freely on the web was used in some portion for the analysis of the result at a significant level of 0.05.

## 3. RESULTS & DISCUSSION

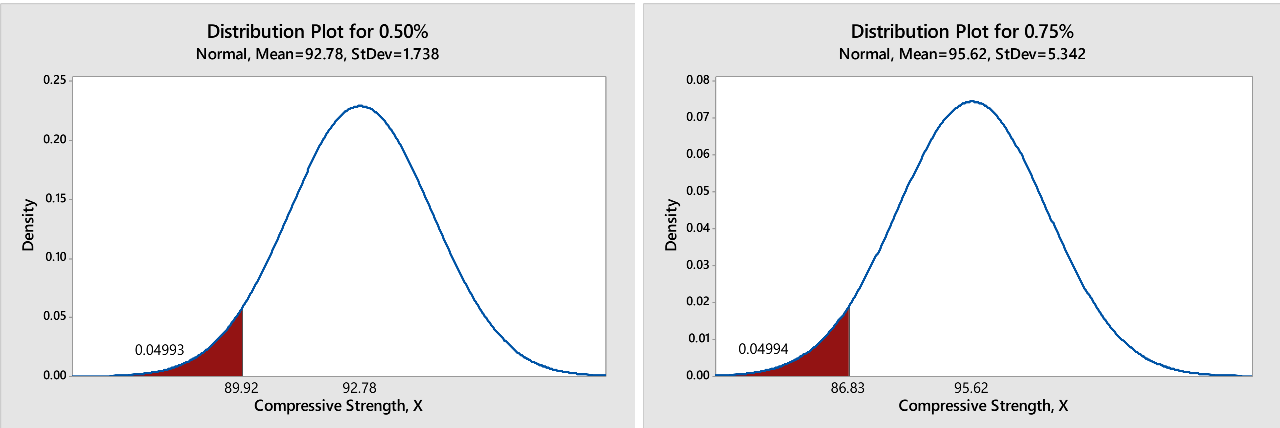
### 3.1 Compressive strength, $f_c$

In terms of $f_c$, it could be seen that when compared with the reference strength (86 MPa) given in Table 4, there is an increase in strength with fiber addition. It is known that the ultimate strength of concrete depends on the behavior of the concrete at the region that is very close to the failure load. In HPC with fiber addition, cracks move parallel to the direction of the loading, randomly distributed fibers intercept the cracks resulting in high carrying capacity of the concrete. Fiber alignment also plays a crucial role where during compaction process, concrete casted in prisms have the affinity to align themselves perpendicular to the casting direction, while their cylindrical counterparts vertically. This has been reported by [38]. However, if the fibers are aligned in the direction of the cracks, the crack will progress parallel to the fiber, and little resistance to the crack growth will be offered.

**Table-5:** Compressive strength results

| Fiber Volume (%) | Mean (MPa) | 5 % of N (MPa) | Range (MPa) |
|---|---|---|---|
| 0.50 | 92.78 | 89.92 | 88.97 – 94.87 |
| 0.75 | 95.62 | 86.83 | 86.89 – 103.36 |
| 1.00 | 97.37 | 91.11 | 88.89 – 102.56 |
| 1.25 | 96.82 | 91.89 | 91.84 – 103.42 |
| 1.50 | 98.82 | 95.38 | 92.53 – 101.77 |

The trend in this result presented in Table 5 and depicted graphically in Fig. 1, shows that 5 % of the population have a compressive strength of less than or equal to 89.92 MPa – 95.38 MPa. This study utilized plastic molds which has been reported to have problems with inducing flaws and deficiencies due to inability to transmit vibration [39]. It might have played a role in reducing the value of the strength, but not as much as when a large specimen is utilized.



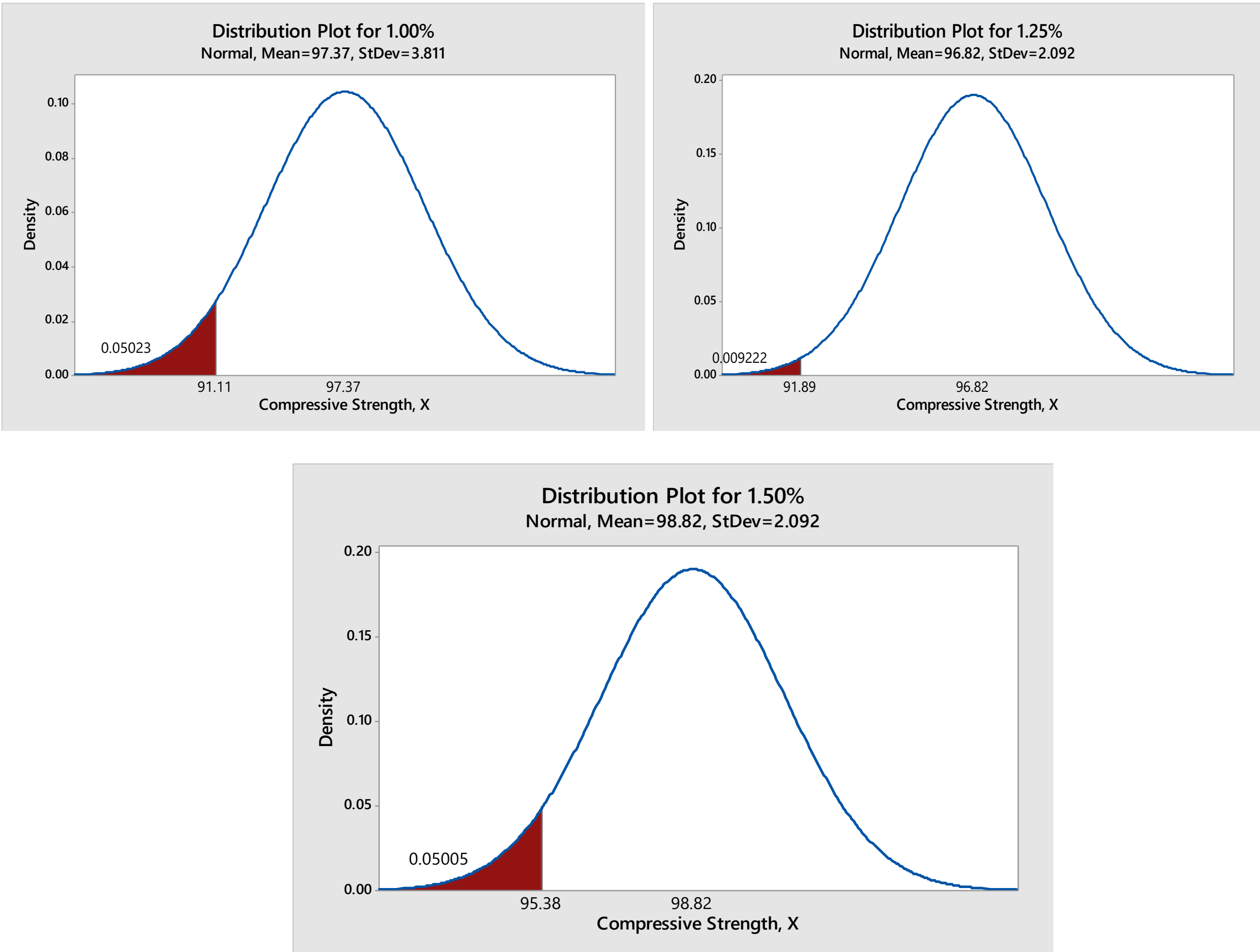



**Fig-1:** Probability plots for 5% and mean compressive strengths

### 3.2 Sample Size & Normality Test

According to Walpole *et al.*, [40] regarding normal approximation in respect of sample size, if N is less than 30 and the population is approximately normal or known to be normal, the sampling distribution of the mean will follow the normal distribution even if the sample size is small, but the distribution becomes better when the sample size grows bigger. In this study, sample size (N) is 50 in each fiber addition level, outlier test was conducted to screen the possibility of extreme values, and the result presented in Table 7.

$$\text{Grubb's test} = \frac{\text{Mean} - (\text{largest or smallest value})}{\text{standard deviation}} \qquad [\text{Eq. 1}]$$

**Table-6:** Grubbs' Outlier Test

| Variable | 0.50% | 0.75% | 1.00% | 1.25% | 1.50% |
|---|---|---|---|---|---|
| **Mean** | 92.78 | 95.62 | 97.37 | 96.82 | 98.82 |
| **StDev** | 1.74 | 5.34 | 3.81 | 2.99 | 2.09 |
| **Minimum** | 88.97 | 86.89 | 88.89 | 91.84 | 92.53 |
| **Maximum** | 94.87 | 103.36 | 102.56 | 103.42 | 101.77 |
| **G** | 2.19 | 1.70 | 2.23 | 2.20 | 3.00 |
| **P-value** | 1.00 | 1.00 | 1.00 | 1.00 | 0.08 |

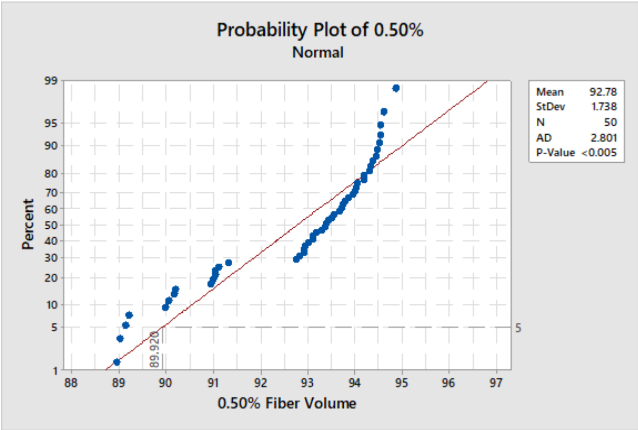
No outlier was detected at the 5% level of significance because P-value is above 0.05 for all volume fraction. The results in Table 6 for the Grubb's outlier test (Eq. 1) that was conducted at a significance level of 0.05 shows no outlier detected at the stated $\alpha$ value. Thereafter, normality test was conducted on N with fiber addition. This entails examining the data to see how closely it fits to the normal distribution by following the fitted normal distribution line. When a data set follows a normal distribution, it follows closely the "ideal normal distribution" straight line with a good fit. In Fig. 2 (a) – (e), it can be seen that the data is swirling or twisting around the normal distribution line, an indication of not an ideal fit. However, there are other ways by which we can determine if the data follows the normal distribution through examining the P-value as well as the Anderson-Darling statistic.

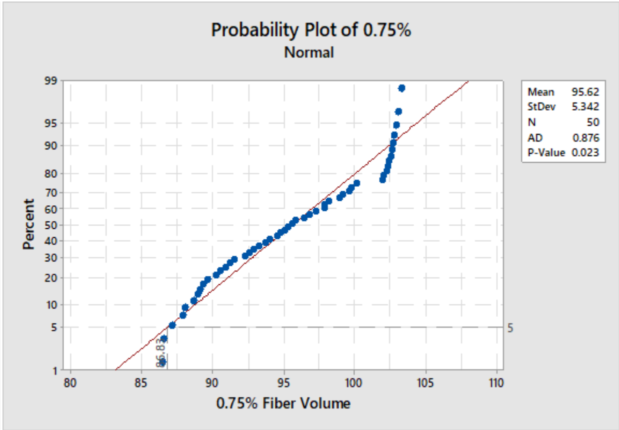
In other to examine these two parameters, we have to make the following assumption or hypothesized that:

Null hypothesis, Ho: Data follow a normal distribution

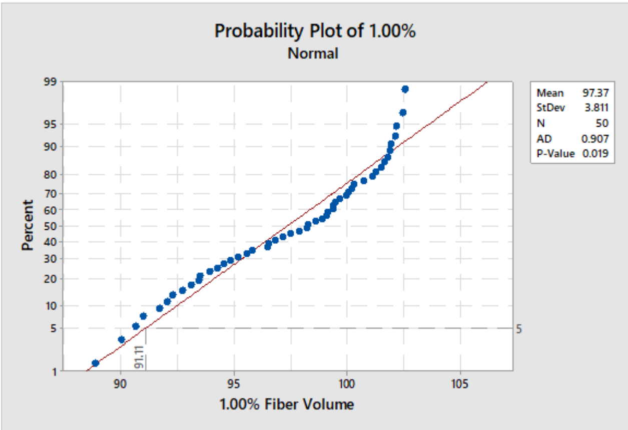
Alternative hypothesis, $H_1$: Not a normal distribution

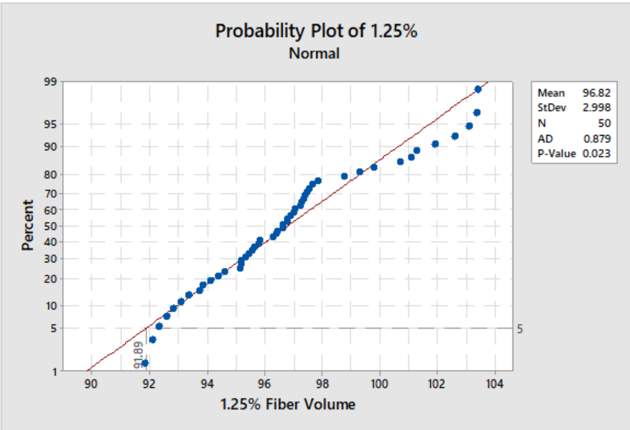
At a significance level of 0.05, all the values were below 0.05 in Fig. 2 with the exception of Fig. 2 (e) that had a P-value of 0.065 which is slightly above the $\alpha$ value. This shows that most of the data do not follow the normal distribution, and we reject the null hypothesis. It is also seen that the Anderson-Darling statistic is high as a consequence of the low P-value.



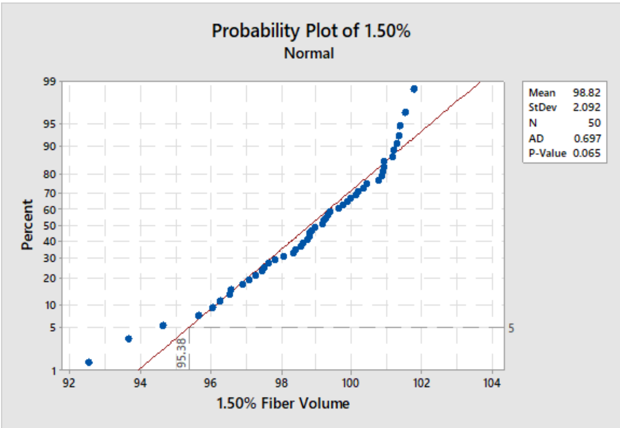




(a) (b)



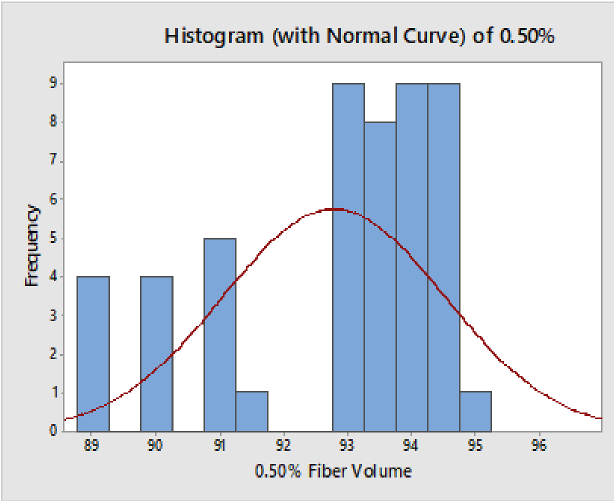

(c) (d)



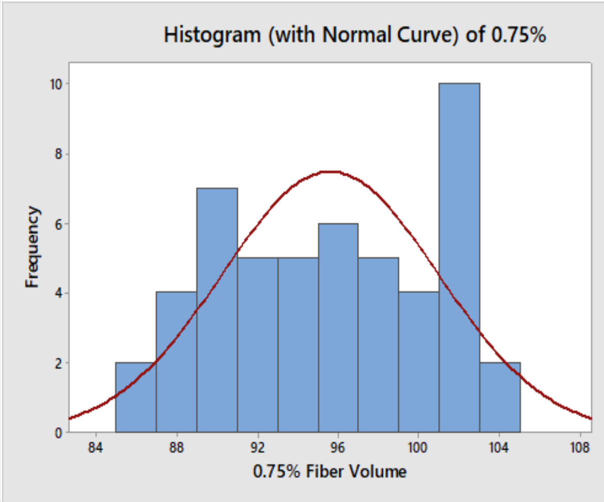

(e)

**Fig-2(a) – (e):** Probability plots for HPC with fiber addition.

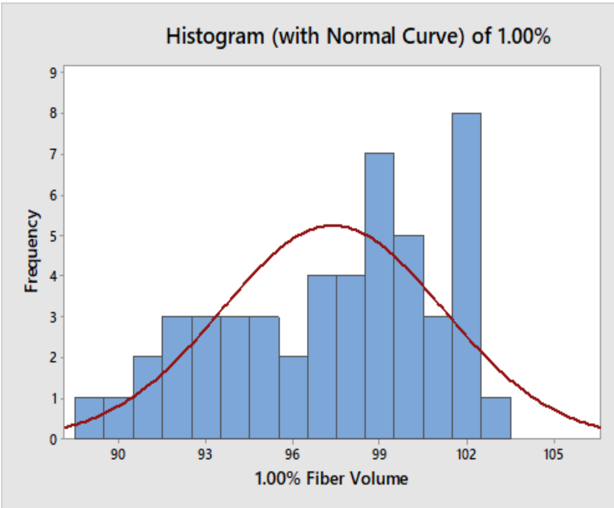
If we fit the normal curve on a frequency histogram, it would be seen that the data would unlikely follow that normal distribution based on the two test conducted earlier where the data do not fit very well on the ideal normal distribution line. Besides, the histogram did not fit into the normal curve as it is displayed in Fig. 3.



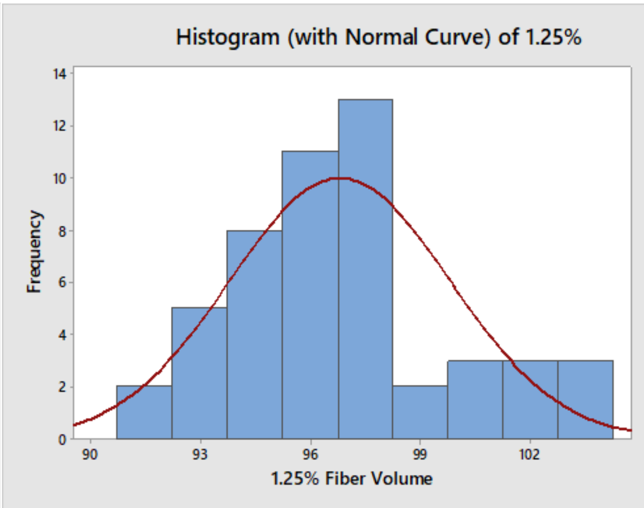

(a)



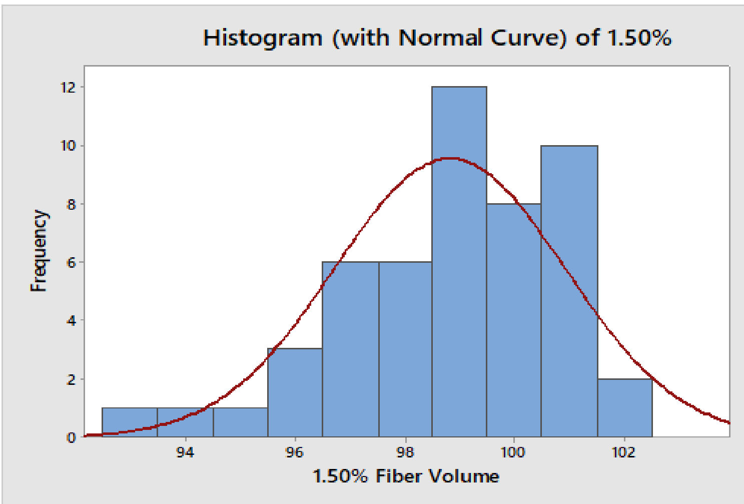

(b)




(c)




(d)




(e)

**Fig-3 (a) – (e):** Frequency histogram with a superimposed normal curve

### 3.3 Basic Statistics

In Table 7, a summary is presented for basic statistics conducted on the set of data. The mean of each N with fiber addition showed an increase with fiber addition level with the exception of 1.25% which showed a decline from the previous addition level. The range of the values is spread over a considerably long values of compressive strength for fiber addition from 0.75% and above with the longest at 0.75%. In 0.50% addition level as seen in Fig. 4, the median value from the Boxplot indicated that it is very close to the third quartile (Q3), the same phenomenon was observed in 1.00% addition level. An indication that the data is distributed in the upper portion of the graph. Percentages of 0.50% and 1.50% had shorter upper whiskers, an indication that it is very close to the maximum values.

Boxplots generally display range and spread of values, as well as agreement or how different the range is. Table 8 in conjunction with Fig. 4 indicates that the result is skewed to the left (negative) with the exception of 1.25% that was positive. Additionally, it could be seen from Fig. 4 that all the Boxplots met at some point in the graph, an indication of overlap that signifies the results have similar characteristics with the variation not that different.

**Table-7:** Basic statistics

| Variable | 0.50% | 0.75% | 1.00% | 1.25% | 1.50% |
|---|---|---|---|---|---|
| **Samples (N)** | 50 | 50 | 50 | 50 | 50 |
| **Mean** | 92.778 | 95.620 | 97.373 | 96.817 | 98.817 |
| **SE Mean** | 0.246 | 0.755 | 0.539 | 0.424 | 0.296 |
| **StDev** | 1.738 | 5.342 | 3.811 | 2.998 | 2.092 |
| **Variance** | 3.019 | 28.536 | 14.521 | 8.986 | 4.377 |
| **Coeff. of Variation** | 1.87 | 5.59 | 3.91 | 3.10 | 2.12 |
| **Minimum** | 88.970 | 86.890 | 88.890 | 91.840 | 92.530 |
| **Q1** | 91.110 | 90.863 | 94.185 | 95.040 | 97.518 |
| **Median** | 93.385 | 95.500 | 98.265 | 96.640 | 99.075 |
| **Q3** | 94.080 | 100.645 | 100.418 | 97.730 | 100.520 |
| **Maximum** | 94.870 | 103.360 | 102.560 | 103.420 | 101.770 |
| **Skewedness** | -0.947 | -0.062 | -0.470 | 0.597 | -0.907 |
| **Kurtosis** | -0.387 | -1.286 | -0.893 | -0.086 | 0.753 |

StDev: Standard Deviation; SE Mean: Standard Error of Mean

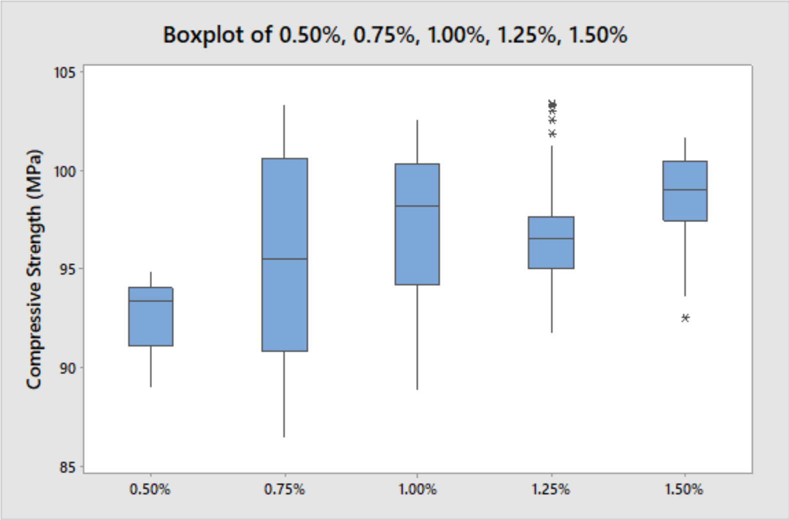
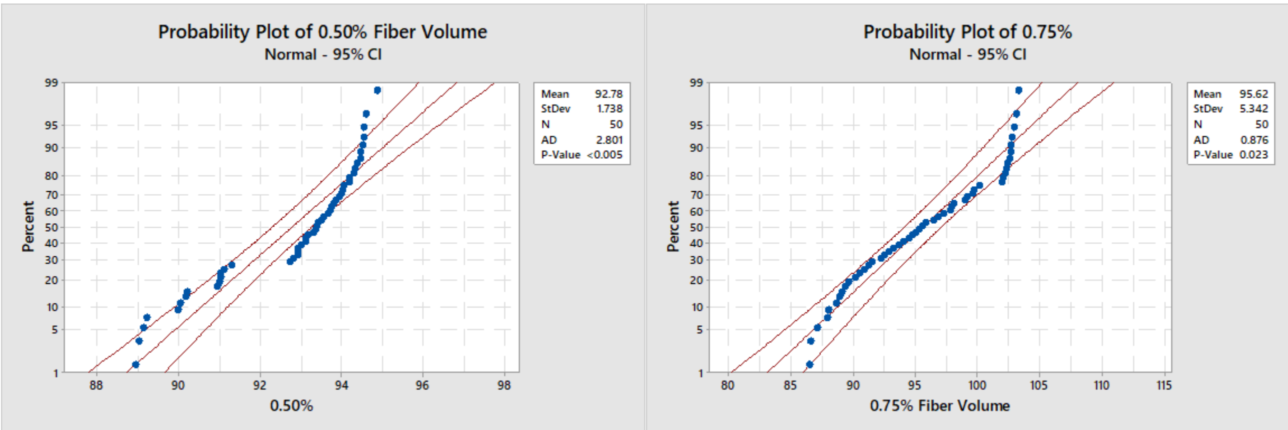



**Fig-4:** Boxplot of the sample data in each fiber level

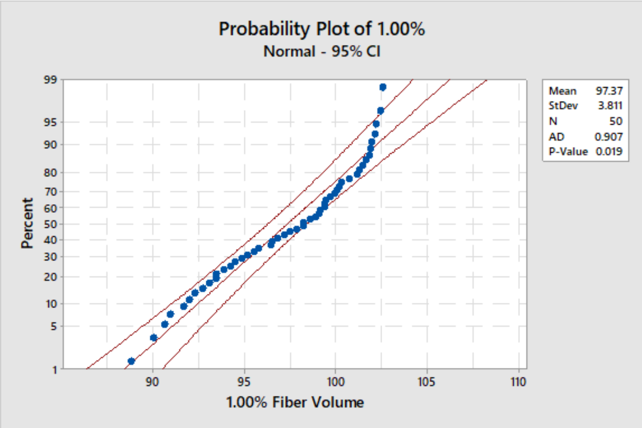
Standard error of the mean which shows the degree of variability of the sample mean presented indicated that 0.75% fiber addition had the highest error at 0.755. This is clearly evident from Fig. 4 where the Boxplot showed a wide range which could be the source of the variability. Standard deviation on the other hand which measured variability in the single sample showed a similar trend observed in standard error of mean.

## 3.4 Confidence Interval (CI) & Probability Plots

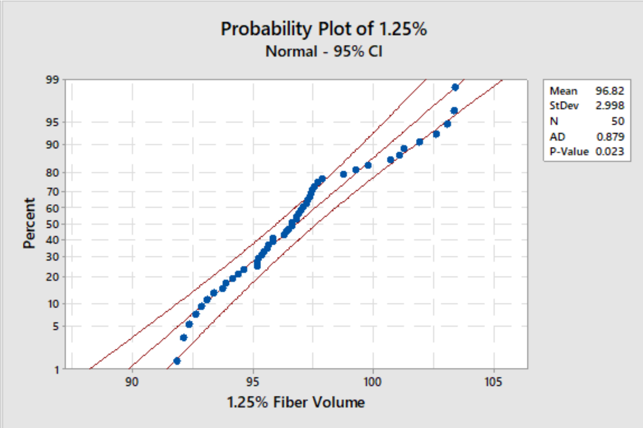
It has already been established that the data is non normal, however, we wish to see if based on the α value of 0.05, does the data fit into the confidence band from the probability plot? In Fig. 5 and Table 8, the probability plot of the confidence interval, and the numerical value is presented. The 95 per cent CI for the mean compressive strength in HPC with fiber addition is as presented. It could be seen that there is an improvement in the CI of the mean strength with fiber addition, except at 1.25%. To correlate the CI with the P-value, as seen earlier, P-value < alpha value (0.05).




(a) (b)




(c)

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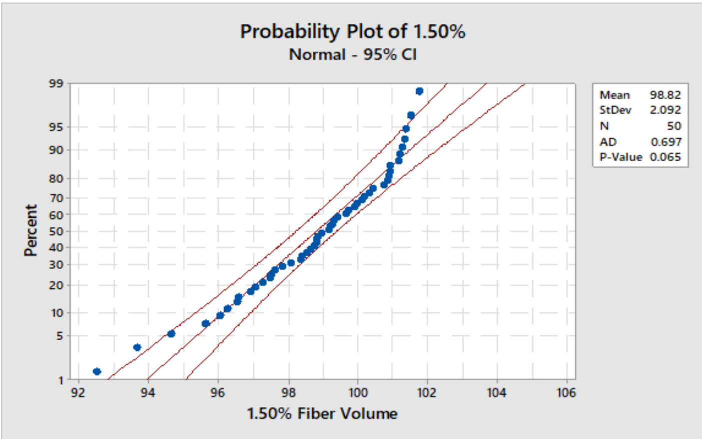



(e)

**Fig-5 (a) – (e):** Confidence Interval of the concrete with fiber addition

**Table-9:** Confidence interval

| Variables | 0.50 % | 0.75 % | 1.00 % | 1.25 % | 1.50 % |
|---|---|---|---|---|---|
| **Mean** | 92.778 | 95.620 | 97.373 | 96.817 | 98.817 |
| **StDev** | 1.738 | 5.342 | 3.811 | 2.998 | 2.092 |
| **SE Mean** | 0.246 | 0.755 | 0.539 | 0.424 | 0.296 |
| **95 % CI for Mean** | (92.284-93.272) | (94.102-97.139) | (96.290-98.456) | (95.965-97.669) | (98.222-99.411) |
| **95 % CI for StDev** | (1.451-2.165) | (4.462-6.657) | (3.183-4.749) | (2.504-3.735) | (1.748-2.607) |
| **Anderson-Darling** | 2.801 | 0.876 | 0.907 | 0.879 | 0.697 |
| **P-Value** | < 0.005 | 0.023 | 0.019 | 0.023 | 0.065 |

The probability plots presented in Fig. 5 for the 95 per cent confidence interval for HPC with fiber addition indicated that most of the samples were in the confidence band except few. Anderson-Darling statistic in all cases was high, with the highest at 0.5% addition. Smaller values of Anderson-Darling indicate that the data follows a normal distribution.

### 3.5 Data Transformation

Having seen that the data does not follow the normal distribution, and of course it is highly unlikely that it will, the idea of data transformation was explored. Data that do not follow normal distribution can be transformed to other distributions such as Lognormal, 3-Parameter, 2-Parameter, Weibull etc. Using Minitab and Eq. 2 - 6 for Johnson transformation function, the Goodness of fit was assessed as presented in Fig. 6 (a) – (e). When the P-value is compared after the transformation in Fig. 6, it could be seen that there is a significant improvement in the value compared with prior to transformation most especially in Johnson Transformation.

The main idea behind the transformation is to find the most appropriate model that best characterize the data since the normal distribution has failed. In a situation where after the transformation, a significant number of model have $\alpha$ value above 0.05, it is advisable to select the model with the lowest P-value that is above 0.05. It is seen that contrary to Fig. 4 where some data were outside the confidence band, in Fig. 6 all the data were within the range of the confidence interval, an indication of the goodness of the fit.

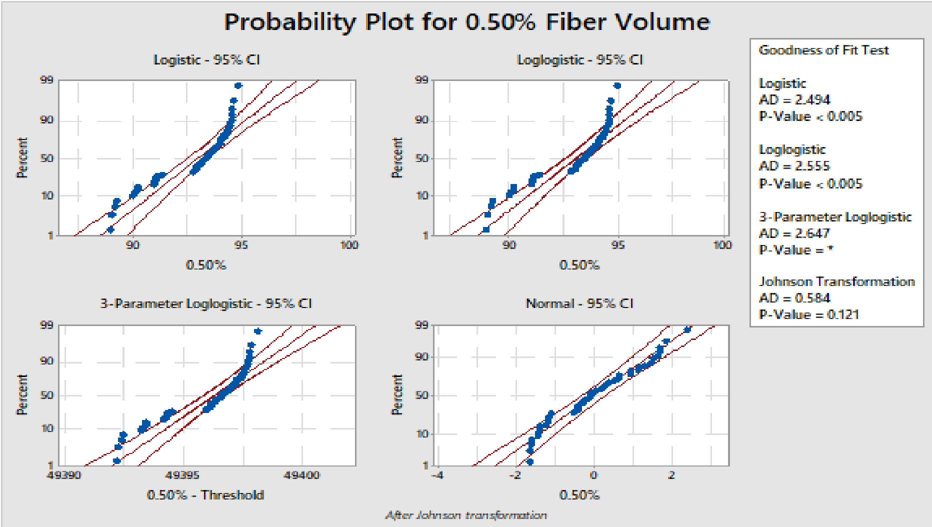
$$0.5\% = (1.59469+0.999006 * \mathrm{ASINH}((X-94.5184)/0.418059))) \qquad [\text{Eq. } 2]$$

$$0.75\% = (-0.103379+0.576821*\mathrm{Ln}((X-86.2355)/(103.837-X))) \qquad [\text{Eq. } 3]$$

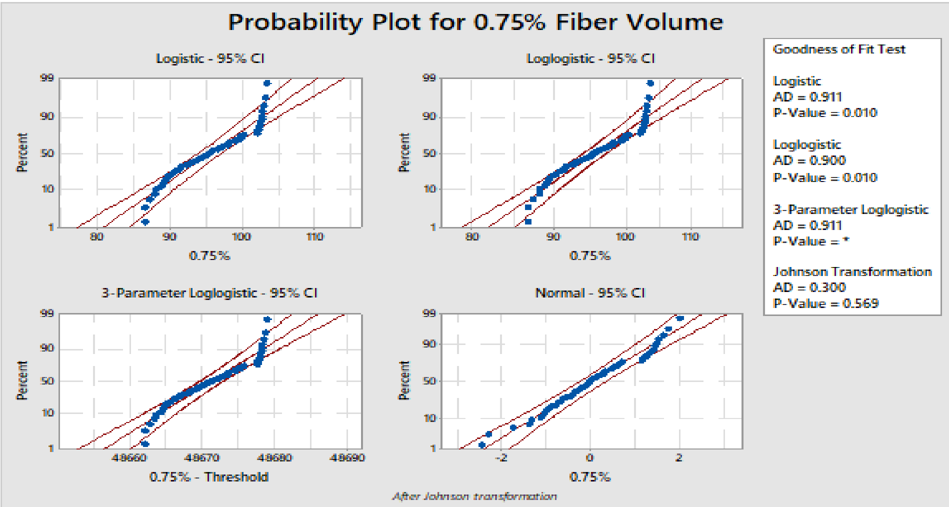
$$1.0\% = (-0.431033+0.616747*\mathrm{Ln}((X-88.4519)/(102.856-X))) \qquad [\text{Eq. } 4]$$

$$1.25\%=(-0.813151+1.65912*\mathrm{ASINH}((X-94.6309)/3.00707))) \qquad [\text{Eq. } 5]$$

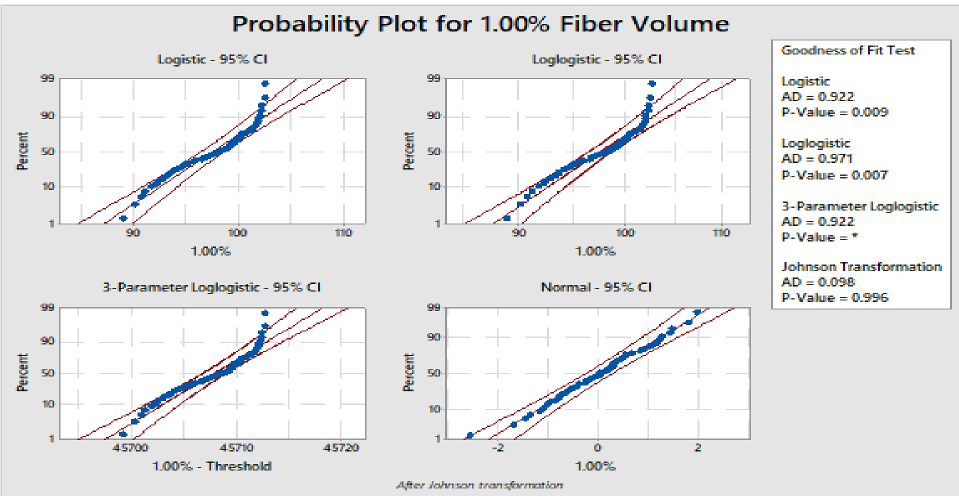
$$1.50\% = (-1.19746+1.09022*\mathrm{Ln}((X-90.0068)/(102.261-X))) \qquad [\text{Eq. } 6]$$


Probability Plot for 0.50% Fiber Volume
Logistic - 95% CI
Loglogistic - 95% CI
3-Parameter Loglogistic - 95% CI
Normal - 95% CI
Percent
0.50%
0.50% - Threshold
After Johnson transformation
Goodness of Fit Test
Logistic
AD = 2.494
P-Value < 0.005
Loglogistic
AD = 2.555
P-Value < 0.005
3-Parameter Loglogistic
AD = 2.647
P-Value = *
Johnson Transformation
AD = 0.584
P-Value = 0.121

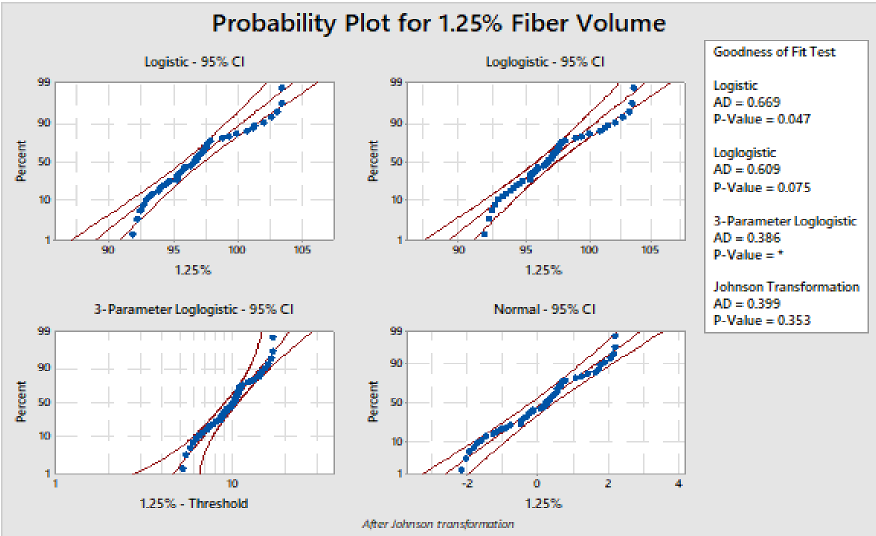
(a)


Probability Plot for 0.75% Fiber Volume
Logistic - 95% CI
Loglogistic - 95% CI
3-Parameter Loglogistic - 95% CI
Normal - 95% CI
Percent
0.75%
0.75% - Threshold
After Johnson transformation
Goodness of Fit Test
Logistic
AD = 0.911
P-Value = 0.010
Loglogistic
AD = 0.900
P-Value = 0.010
3-Parameter Loglogistic
AD = 0.911
P-Value = *
Johnson Transformation
AD = 0.300
P-Value = 0.569

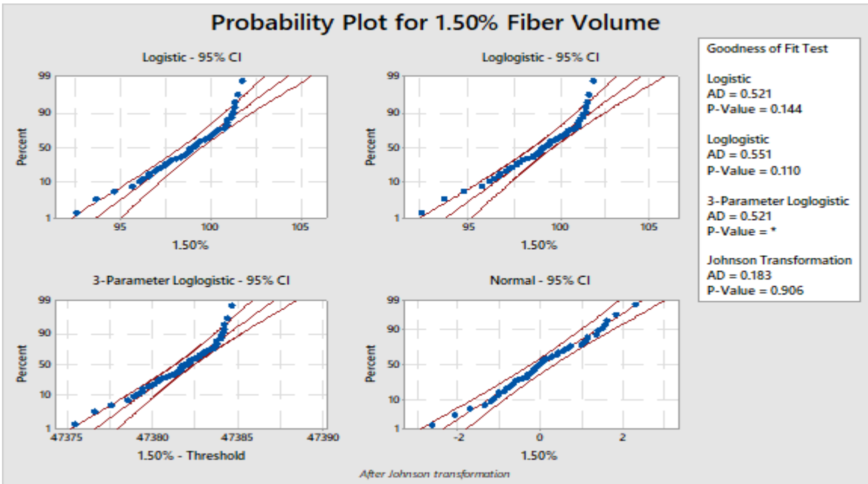
(b)


Probability Plot for 1.00% Fiber Volume
Logistic - 95% CI
Loglogistic - 95% CI
3-Parameter Loglogistic - 95% CI
Normal - 95% CI
Percent
1.00%
1.00% - Threshold
After Johnson transformation
Goodness of Fit Test
Logistic
AD = 0.922
P-Value = 0.009
Loglogistic
AD = 0.971
P-Value = 0.007
3-Parameter Loglogistic
AD = 0.922
P-Value = *
Johnson Transformation
AD = 0.098
P-Value = 0.996

(c)




(d)




(e)

**Fig-6 (a) – (e):** Goodness of fit test after Johnson Transformation

## 4. CONCLUSIONS

A statistical evaluation was conducted on HPC with steel fiber addition and the following conclusions have been reached:

i. Mean compressive strength increase is observed with respect to increase in fiber addition when compared with reference sample, an indication of the advantage of using a large sample size to determine the mean.

ii. The range of values for the set of data appears to be wide especially as the fiber addition increased significantly. It is also seen that 5% of the sample in concrete with different fiber addition will attain a compressive strength of 89.92, 86.83, 91.11, 91.89 & 95.38 MPa respectively.

iii. Normality test and screening test conducted shows that the samples do not follow the normal distribution. This is unlikely because the P-value was very low (below the significance level), Anderson-darling is very high, and the frequency histogram do not fit into the bell-shaped normal curve.

iv. Errors associated with the mean indicates the highest variability was highest at 0.75% fiber addition. This could be associated with the data that swirl around the ideal probability distribution plot.

v. Goodness of fit test attempted by transformation of the data indicates that the P-value is above the significance level after transformation.

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Appendix: Compressive strength results

| | Compressive Strength (MPa) | | | | |
|---|---|---|---|---|---|
| | **0.50 %** | **0.75 %** | **1.00 %** | **1.25 %** | **1.50 %** |
| 1 | 88.97 | 86.52 | 88.89 | 91.84 | 92.53 |
| 2 | 89.03 | 86.62 | 90.06 | 92.11 | 93.67 |
| 3 | 89.15 | 87.16 | 90.68 | 92.37 | 94.66 |
| 4 | 89.23 | 87.97 | 91.02 | 92.62 | 95.65 |
| 5 | 89.99 | 88.09 | 91.72 | 92.85 | 96.06 |
| 6 | 90.06 | 88.71 | 92.06 | 93.12 | 96.26 |
| 7 | 90.19 | 89 | 92.32 | 93.37 | 96.56 |
| 8 | 90.21 | 89.19 | 92.75 | 93.75 | 96.58 |
| 9 | 90.95 | 89.4 | 93.12 | 93.89 | 96.92 |
| 10 | 91 | 89.66 | 93.48 | 94.15 | 97.09 |
| 11 | 91.05 | 90.25 | 93.51 | 94.39 | 97.28 |
| 12 | 91.05 | 90.6 | 93.93 | 94.65 | 97.48 |
| 13 | 91.13 | 90.95 | 94.27 | 95.17 | 97.53 |
| 14 | 91.32 | 91.27 | 94.56 | 95.2 | 97.64 |
| 15 | 92.75 | 91.58 | 94.89 | 95.22 | 97.84 |
| 16 | 92.82 | 92.29 | 95.18 | 95.37 | 98.08 |
| 17 | 92.92 | 92.61 | 95.57 | 95.47 | 98.36 |
| 18 | 92.92 | 92.89 | 95.82 | 95.6 | 98.42 |
| 19 | 92.94 | 93.3 | 96.5 | 95.66 | 98.56 |

| | | | | | |
|---|---|---|---|---|---|
| 20 | 93.01 | 93.77 | 96.53 | 95.82 | 98.64 |
| 21 | 93.12 | 94.06 | 96.86 | 95.84 | 98.75 |
| 22 | 93.12 | 94.56 | 97.19 | 96.29 | 98.81 |
| 23 | 93.18 | 94.83 | 97.5 | 96.42 | 98.82 |
| 24 | 93.3 | 95.11 | 97.88 | 96.47 | 98.87 |
| 25 | 93.37 | 95.36 | 98.26 | 96.63 | 98.98 |
| 26 | 93.4 | 95.64 | 98.27 | 96.65 | 99.17 |
| 27 | 93.43 | 95.9 | 98.63 | 96.81 | 99.21 |
| 28 | 93.5 | 96.5 | 98.92 | 96.81 | 99.28 |
| 29 | 93.56 | 96.87 | 99.13 | 96.93 | 99.32 |
| 30 | 93.68 | 97.29 | 99.16 | 97.02 | 99.41 |
| 31 | 93.71 | 97.88 | 99.4 | 97.08 | 99.65 |
| 32 | 93.75 | 97.94 | 99.42 | 97.24 | 99.75 |
| 33 | 93.8 | 98.17 | 99.49 | 97.28 | 99.9 |
| 34 | 93.87 | 98.99 | 99.7 | 97.36 | 99.97 |
| 35 | 93.95 | 99.17 | 99.97 | 97.42 | 100.13 |
| 36 | 94 | 99.66 | 100.05 | 97.49 | 100.2 |
| 37 | 94.03 | 99.77 | 100.23 | 97.57 | 100.33 |
| 38 | 94.04 | 100.2 | 100.31 | 97.68 | 100.44 |
| 39 | 94.2 | 101.98 | 100.74 | 97.88 | 100.76 |
| 40 | 94.2 | 102.07 | 101.15 | 98.76 | 100.86 |
| 41 | 94.3 | 102.26 | 101.3 | 99.29 | 100.89 |
| 42 | 94.33 | 102.33 | 101.51 | 99.81 | 100.93 |
| 43 | 94.38 | 102.4 | 101.64 | 100.72 | 100.94 |
| 44 | 94.45 | 102.58 | 101.82 | 101.1 | 101.16 |
| 45 | 94.47 | 102.66 | 101.91 | 101.29 | 101.19 |
| 46 | 94.52 | 102.72 | 101.96 | 101.92 | 101.29 |
| 47 | 94.53 | 102.82 | 102.16 | 102.61 | 101.34 |
| 48 | 94.54 | 102.98 | 102.21 | 103.09 | 101.37 |
| 49 | 94.6 | 103.13 | 102.46 | 103.36 | 101.53 |
| 50 | 94.87 | 103.36 | 102.56 | 103.42 | 101.77 |